DEPARTMENT OF THE ARMY CORPS OF ENGINEERS

BEACH EROSION BOARD
OFFICE OF THE CHIEF OF ENGINEERS

THE EFFECT OF FETCH WIDTH ON WAVE GENERATION

TECHNICAL MEMORANDUM NO. 70

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FOREWORD

This paper presents a method for introducing the effect of "fetch width" into the prediction of waves generated by wind. It has been generally recognized that waves are generated not only in the direction of the generating wind but also over a rather considerable angle to this wind direction. This condition has been tacitly recognized in wave prediction formulas for ocean conditions where fetch widths are generally large; so far as is known, no method of assessing the effect of fetch width on wave generation in waters of limited width has been presented prior to this paper.

The author of the paper, Thorndike Saville, Jr. of the Research Staff of the Beach Erosion Board, originated the concepts for adjusting for fetch width presented in the paper. These concepts were formalized during a 4-week work conference of representatives of various Corps of Engineers offices held at the Beach Erosion Board on the analysis of waves in inland reservoirs. The conference was arranged by the Hydrology and Hydraulics Branch of the Office of the Chief of Engineers. Conferees were E. W. McClendon and A. M. Franklin of the Missouri River Division, and G. C. Kelley and P. Veale of the Tulsa District; part time conferees, in addition to the author, were C. L. Bretschneider of Texas A. & M. College; A. L. Cochran, R. N. Wilson and C. P. Pletcher of the Office, Chief of Engineers and J. M. Caldwell of the Beach Erosion Board. The work done on this paper was supported by the Civil Works Investigation Program of the Office of the Chief of Engineers under project CW166, "Study of Waves and Wind Tides in Shallow Water".

The present report was prepared by Thorndike Saville, Jr., Chief of the Special Projects Branch of the Research Division of the Beach Erosion Board under the supervision of J. M. Caldwell, Chief of the Division. At the time the report was prepared, the technical staff of the Board was under the general supervision of Colonel W. P. Trower, President of the Board, Colonel E. A. Hansen, Resident Member, and R. O. Eaton, Chief Technical Assistant. The report was edited for publication by A. C. Rayner, Chief, Project Development Division. Views and conclusions stated herein are not necessarily those of the Beach Erosion Board

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THE EFFECT OF FETCH WIDTH ON WAVE GENERATION

by
Thorndike Saville, Jr.
Beach Erosion Board

The effect of fetch width in limiting wave growth in a generating area has long been recognized, but has generally been neglected since, for the generation of waves in the ocean, the vast majority of fetches have widths of the same order of magnitude as the lengths; in such cases the limiting effect of the fetch width has been considered as being very minor. However, in considering wave generation in inland waters (lakes and reservoirs) the fetches are limited not by the (generally) large bounds of the meteorological disturbances (as in the ocean) but by the land forms surrounding the body of water; in these cases, fetches of rather great length in comparison to their width are frequently observed, and the width effect of the fetch may become quite important, resulting in the generation of waves materially lower than those that would be expected from the same generating conditions over more open waters.

It is generally recognized that, in a generating area, waves are generated not only in the immediate direction of the wind, but also at various angles with the wind direction. This wave variability has been shown both from aerial photographs, and by visual and instrumental observations. The energy reaching, and hence the wave characteristics that are measured at a particular point at the end of the fetch, are therefore dependent not only on the spectral components generated (and propagated) in a direction coincident with the wind direction but also on those components generated in directions at angles to the wind direction; the actual observed wave characteristics at a point will result from the summing up of all these components. This may be seen in Figure 1 where Point A is a point at the end of a particular idealized fetch; the wind direction, as indicated by the arrow, is along line OA. Point A, however, is receiving significant amounts of energy from directions at an angle to the wind, as, for example, along the lines 1A to 9A radiating out from A to the rear edge of the fetch. If now the fetch width is limited (as shown in the figure) to the distance BC by the lines BB' and CC', then the distance over which wave components travelling at an angle to the wind greater than that denoted by about lines 3A, will be reduced, and the wave components reaching point A along lines 4A through 9A will be less than these components would have been had the fetch width been unlimited. It may be assumed that the effectiveness of any one directional segment of the fetch in producing waves at point A is the ratio of the length of the fetch segment as limited by the width to its full length, were the fetch width unlimited. For example, line 9A (which may be assumed to represent a certain directional fetch segment) is

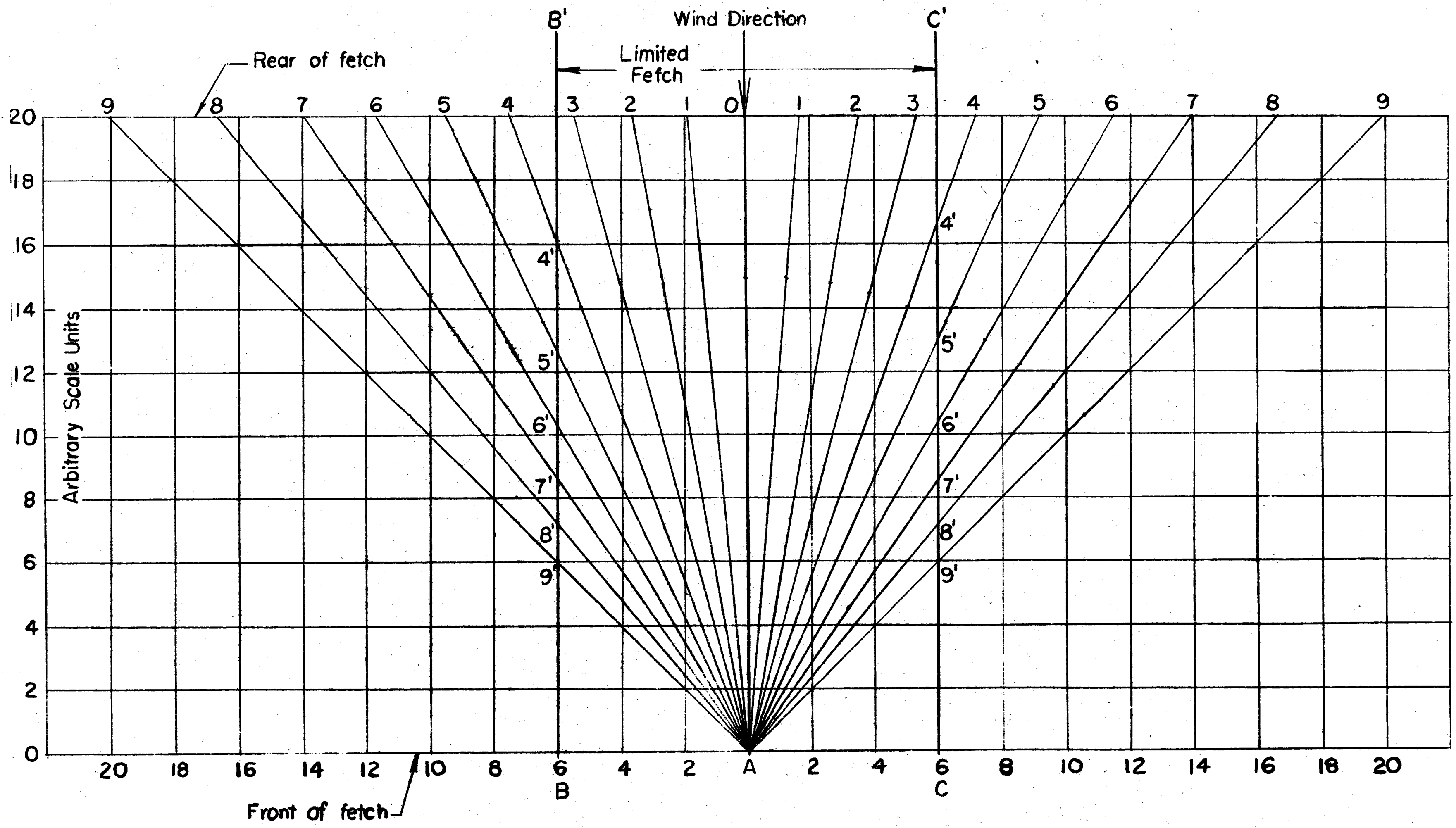

FIGURE I IDEALIZED RECTANGULAR FETCH

28.3 units long; however the portion of this line within the limited fetch is only 8.5 units long. The effectiveness of that portion of the fetch toward point A in the direction along line 9A is then the ratio of these two lengths, 8.5/28.3, or 0.3. Similarly the effectiveness of the fetch segments in other directions may also be computed. The values for the particular case shown in Figure 1 are tabulated below.

TABLE 1

	Fetch Segment (Line) Length				
Line	Unlimited Width	Limited Width	Fetch Segment Effectiveness	Number of Equal Sized Segments	Effectiveness times Number of Segments
0A	20.0	20.0	20/20 = 1.0	1	1.0
1A	20.0	20.0	20/20 = 1.0	2	2.0
2A	20.0	20.0	20/20 = 1.0	2	2.0
3A	20.0	20.0	20/20 = 1.0	2	2.0
4A	21.3	17.5	17.5/21.3 = 0.82	2	1.64
5A	22.1	14.2	14.2/22.1 = 0.64	2	1.28
6A	23.1	12.0	12.0/23.1 = 0.52	2	1.04
7A	24.4	10.5	10.5/24.4 = 0.43	2	0.86
8A	26.1	9.3	9.3/26.1 = 0.36	2	0.72
9A	28.3	8.5	8.5/28.3 = 0.30	2	0.60
Totals				19	13.14

The average effectiveness of the entire fetch may then be obtained by averaging the effectiveness of the individual segments. In this particular case, the lines were drawn at 5-degree increments, and hence the segments represented by the various lines are of equal (angular) size. In the case shown, the sum of the effectiveness values of the individual segments is 13.14, and there were 19 segments. The average effectiveness of the fetch then is 13.14/19 or about 0.70. This would mean that the waves observed at point A would have values corresponding to generation in a fetch that was only about 70 percent as long as the actual fetch lengths due to the effect of limited width, for this particular width-length ratio (0.6).

This 0.7 value of fetch effectiveness was obtained by measuring the actual length of each fetch segment. However, it is not the actual lengths that are important, but the ratio of the two lengths (those for fetches of unlimited width and limited width). In the practical application, this may be obtained much more easily by measuring the projection of these lengths along the wind axis, and computing ratios from these. As, for example, with line 9A, the projection on the wind axis (0A) of the full length (unlimited width) is 20 units, and that of line 9'A is 6 units (the vertical scale reading at the point of intersection of line 9A with BB' and

CC', the fetch width limits); the ratio is 6/20, or 0.3, the same as obtained before. Similarly, values for the other line segments may be obtained, as given in Table 2.

It may be seen that the same values of effectiveness are obtained for the individual fetch segments, and that the same average effectiveness of 13.14/19 = 0.70 results. However, it will be noted that the projected fetch length for the unlimited fetch was always the same (20 units) regardless of which line was used. This means that it is not necessary to compute the effectiveness for each individual fetch segment, but that an average effectiveness value for the fetch as a whole may be obtained directly from the measurements of (projected) fetch length. In the case shown, the sum of the projected fetch lengths (for the limited-width fetch) is 262.8; the effective fetch length may be determined by dividing this by the number of segments as 262.8/19 = 13.9 units, and the average effectiveness of the fetch as 262.8/19 x 20 = 0.7. This may be seen to be identical with that determined before.

TABLE 2

Line	Fetch Segment (Line) Length Projected on Wind Axis		Fetch Segment Effectiveness	No. of Segments	Effectiveness times No. of Segments	Limited-Width Fetch Fetch Length times No. of Segments
	Unlimited Width	Limited Width				
0A	20.0	20.0	20/20 = 1.0	1	1.0	20.0
1A	20.0	20.0	20/20 = 1.0	2	2.0	40.0
2A	20.0	20.0	20/20 = 1.0	2	2.0	40.0
3A	20.0	20.0	20/20 = 1.0	2	2.0	40.0
4A	20.0	16.5	16.5/20 = 0.82	2	1.64	33.0
5A	20.0	12.8	12.8/20 = 0.64	2	1.28	25.6
6A	20.0	10.4	10.4/20 = 0.52	2	1.04	20.8
7A	20.0	8.6	8.6/20 = 0.43	2	0.86	17.2
8A	20.0	7.1	7.1/20 = 0.36	2	0.72	14.2
9A	20.0	6.0	6.0/20 = 0.30	2	0.60	12.0
Totals				19	13.14	262.8

The method given above of determining the effective fetch assumes that, for a fetch of unlimited width, point A receives equal amounts of wave energy from each equal-angle directional segment. This is the assumption that has generally been made in practice for using the Sverdrup-Munk forecasting curves, or the Bretschneider revisions thereof, where the general application is to assume that full values of wave characteristics are generated in directions up to an angle of 30 or 45 degrees with the wind direction, and that none is generated beyond that angle. This essentially means that the actual effectiveness of the wind in raising waves is 100 percent up to an angle of 30 or 45 degrees with the wind direction, and zero percent effective for angles greater than that. The case shown in Figure 1 is for winds up to 45 degrees. Values of fetch effectiveness for various values of the fetch width-length ratio have been determined using 1-degree increments for a rectangular fetch for a 100 percent effective wind over both 60 and 90 degrees of fetch (i.e. 30 and 45 degrees to either side of the wind direction) and are plotted as the dashed lines in Figure 2.

Any other type of variation of wind effectiveness with angle to the wind might be assumed, and the same method of analysis applied to it. Computations of wind set-up generally assume that the stress applied to the water surface (wind effectiveness) varies with the cosine of the angle with the wind direction. This same variation has also been suggested as representing a more correct picture of the wind effectiveness in wave generation. The same type of analysis as before may be made for the case shown in Figure 1. For example, line 9-9'A is at a 45-degree angle to the wind; the fetch effectiveness of the fetch segment represented by line 9'A or 9A was determined as 0.30. The cosine of 45 degrees is 0.707, and this value represents the wind effectiveness for this particular fetch segment. The total effectiveness (both fetch and wind) of the fetch segment 9-9'A is then the product of these two values, or 0.3 x 0.707 = 0.21. Values may be computed similarly for the entire condition of Figure 1, as tabulated in Table 3. The average effectiveness of the entire fetch is then the sum of these computed segment effectivenesses divided by the sum of the values of wind effectiveness for each segment. (That the divisor is the sum of the wind effectivenesses rather than the total number of segments (19) is obvious since if the fetch width is unlimited, the average effectiveness for the whole fetch must be equal to 1. Actually this merely weights the value of each segment according to its wind effectiveness -- i.e., there are still the same number of segments, but they no longer have an equal effect on point A). In this particular case, if the fetch effectivenesses are taken from Table 1 or 2, the sum of the total effectivenesses may be computed as 12.18, and the average effectiveness of the entire fetch is 12.18/16.90 = 0.72.

Again, in the practical computation of these values, a somewhat easier method may be used since the projected length for each line segment for the fetch of unlimited width is always the same (here, 20 units). Instead of actually computing the fetch (segment) effectiveness (the ratio of the two lengths), the projected length for the limited-width fetch is multiplied directly by the wind effectiveness

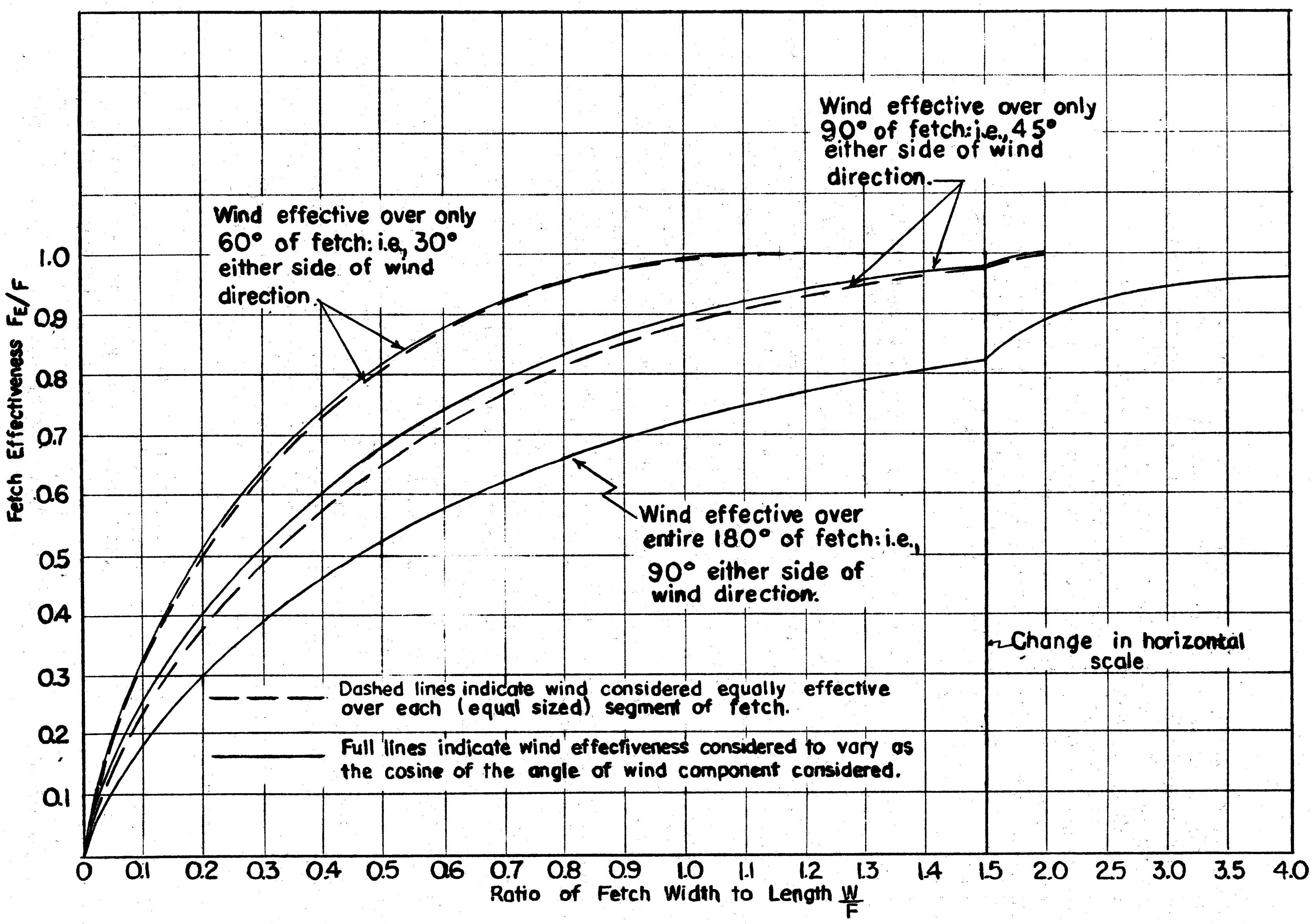

FIGURE 2. RELATION OF EFFECTIVE FETCH TO WIDTH—LENGTH RATIO FOR RECTANGULAR FETCHES

(cos α), and these values are then summed. This summation may be divided by the unlimited-width fetch length (here 20 units) to obtain a summed value of total effectiveness (243.72/20 = 12.19), by the sum of the wind effectivenesses to obtain the value of effect fetch length (243.72/16.902 = 14.4), or by both of these to obtain the average effectiveness of the entire fetch (243.72/20 x 16.902 = 0.72). In this way, only columns 1, 2, 5, and 6 of Table 3 are necessary to make the desired computations.

TABLE 3

Line	(1) Angle to Wind (α)	(2) Wind Effectiveness (cos α)	(3) Fetch Effectiveness	(4) Total Effectiveness	(5) Projected Length Limited Width	(6) Projected Length times cos α
9A	45	.707	0.30	0.21	6.0	4.24
8A	40	.766	0.36	0.28	7.2	5.52
7A	35	.819	0.43	0.35	8.6	7.04
6A	30	.866	0.52	0.45	10.4	9.01
5A	25	.906	0.64	0.58	12.8	11.60
4A	20	.940	0.82	0.77	16.5	15.51
3A	15	.966	1.00	0.97	20.0	19.32
2A	10	.985	1.00	0.98	20.0	19.70
1A	5	.996	1.00	1.00	20.0	19.92
0A	0	1.000	1.00	1.00	20.0	20.0
1A	5	.996	1.00	1.00	20.0	19.92
2A	10	.985	1.00	0.98	20.0	19.70
3A	15	.966	1.00	0.97	20.0	19.32
4A	20	.940	0.82	0.77	16.5	15.51
5A	25	.906	0.64	0.58	12.8	11.60
6A	30	.866	0.52	0.45	10.4	9.01
7A	35	.819	0.43	0.35	8.6	7.04
8A	40	.766	0.36	0.28	7.2	5.52
9A	45	.707	0.30	0.21	6.0	4.24
Totals		16.902		12.18		243.72

Values of fetch effectiveness computed in this way using 1-degree increments for rectangular fetches for various fetch width-length ratios are shown by the solid lines in Figure 2. Values computed in this way are shown for angular spreads about the wind direction of 60, 90, and 180 degrees (i.e., the wind being effective to 30, 45 and 90 degrees to either side of the wind direction).

The curves shown in Figure 2 represent the most usual assumptions that have been made as to wind distribution conditions for wave generation; however, any other assumption could be made, and the method applied in a manner similar to that shown. It might be noted that very little difference is observed between the curves obtained by the two assumptions as to wind direction (i.e., equal wind effect for each angular segment, or wind effect varying as the cosine of the angle with the wind) as long as the same range of effectiveness (30 or 45 degrees here) is used. This is true for the idealized rectangular fetches used herein. However, the method is equally applicable to the actual irregular fetches of natural lakes, but enough comparisons have not been made for these to be certain that essentially no difference between the two would result, although this is suspected to be the case.

The values obtained by using the cosine wind distribution over the entire 180 degrees of fetch are probably theoretically more sound than any of the others obtained. It will be noted though, that for this wind distribution, for a fetch width-length ratio of 1 (i.e., fetch width is equal to fetch length), the fetch effectiveness is only 0.72; and, indeed, does not even quite reach 90 percent for a fetch whose width is twice the length. Hence, in the practical case of use of these effective fetch curves with the wave generation or forecasting curves already in use, the curve based on the cosine wind distribution over the whole 180 degree sector would lead to values that were too low; for although this curve may be theoretically more correct, the actual forecasting curves were derived without consideration of any effect of fetch width. Since the empirical data used in obtaining the present forecasting curves are largely ocean data where fetch width-length ratios generally range between about 1 and 2, the 100 percent effective fetch value (for use with those curves) should be somewhere within this range. Actually, the forecasting curves already derived possibly should be re-examined in view of this fetch-width limitation, and new curves drawn, but this is a time consuming job, and in view of the accuracy of the observations and determined weather conditions, probably is not warranted.

In this connection, analysis by E. W. McClendon, A. M. Franklin, G. C. Kelley and P. Veale, of several years of wave and wind data taken by the Corps of Engineers at Denison and Fort Peck Reservoirs has shown that use of the cosine wind variation over only 90 degrees of fetch (45 degrees either side of the wind direction) yields results that conform very closely to the forecasting curves presented by Bretschneider. Messrs. McClendon, Franklin, Kelly, Veale, and others are presently preparing a report on this subject, and the generation of waves in inland reservoirs as a whole, which is scheduled for publication as a Corps of Engineers report sometime in 1955.

Actually, some theoretical support for the use of a 90-degree sector only may be given if the effect of sheltering is considered. The waves generated and travelling in directions at an angle to the wind will be of lower height and period, and may be expected to be sheltered somewhat from the wind effect by the higher waves travelling more nearly in the direction of the wind. With a large angle to the wind, this sheltering effect may become quite considerable, and essentially prohibit growth of the wave components travelling in directions greater than some particular angle with the wind (say 45 degrees). Not enough is known today, however, about such sheltering effects to allow any definite determination to be made.

To sum up, a method of determining the effect of fetch width on wave generation has been presented, primarily for use in predicting wave characteristics in inland waters. Several curves were given representing several assumptions as to the variation of wind strength or effectiveness in the fetch, though some evidence seems to indicate that use of the curves based on wind strength varying as the cosine of the angle up to 45 degrees and being zero from there out, gives results which most nearly conform with the wave forecasting curves already in use. It should be noted that while the curves presented are for idealized rectangular fetches (which will probably be sufficient for many lake, reservoir or ocean cases) the actual fetch shape can be used and the same method of determination of effectiveness applied. It is interesting to note that use of this method with actual fetch shapes where the fetch is limited in a direction coincident with the wind direction but stretches out longer in directions at an angle to the wind, may result in an effective fetch length which is actually greater than the (limited) distance in the direction of the wind due to the added angular components.